

# UPLIFT OF ANCHOR PLATES IN SAND

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**ABSTRACT:** An investigation into the vertical uplift of anchor plates in a cohesionless medium is described. Factors investigated in relation to the load-displacement response were the size and shape of plate, depth of embedment, sand density, and plate surface roughness. The results of laboratory tests are presented, together with equilibrium and limit analysis methods of predicting the ultimate resistance. Comparisons are made between the laboratory test results, the developed analyses, and existing theories. It is shown that the different theoretical solutions examined predict a relatively wide range of ultimate uplift resistance with no one approach showing good agreement with the experimental evidence for all cases examined.

## INTRODUCTION

There are numerous forms of soil anchor described in the literature (1,3,6,10-12,15,19,23,25-30). This paper is concerned with the behavior of those anchors that are used to resist vertical uplift loads and that develop a passive form of resistance. To this end, a laboratory test program has been carried out to examine the uplift response of plate anchors of various shapes embedded in cohesionless media. Although there are no fully adequate substitutes for full-scale field tests, tests at laboratory scale have the advantage of allowing a close control of at least some of the variables encountered in practice. In this way trends and behavior patterns observed in the laboratory can be of value in developing an understanding of performance at larger scales, and mathematical analyses developed in conjunction with laboratory testing may thus be of value in practice.

A number of theories for estimating the passive uplift resistance of anchors and the allied problem of the uplift of footings in a cohesionless medium have been published. Of those considered in this paper, the simplest is Mors's method (17). Essentially this is a modified "cone of earth" solution; in his analysis he ignores the soil shearing resistance, although the volume of the affected soil is altered empirically according to the type of soil involved.

A more practical procedure is that of Meyerhof and Adams (16). Using simplifying assumptions and comparisons with experimental data they produced expressions for the vertical uplift resistance of strip, rectangular, and circular footings at both shallow and greater depths.

Balla (2) adopted a more complex approach for determining the breakout of mushroom (underreamed) circular foundations at shallow depth. He assumed a circular failure surface and used Kotter's equation to

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describe the distribution of shearing resistance on that surface. However, it should be noted that in examining the vertical equilibrium of the system, Balla ignored the presence of normal stresses on the failure surface and only resolved the shear stress components in the vertical direction. Adopting a similar approach to Balla but using a logarithmic spiral failure surface, Khadilkar, et al. (9) developed an analysis for the uplift of anchor foundations. However, their analysis does not produce frictional resistance forces on the failure surface in the required direction and is thus invalid.

Matsuo (14) assumed the failure surface to be approximated by a logarithmic spiral terminating in a passive wedge zone near the soil surface. Using Kotter's equation to determine the shearing stresses, he developed an approximate theory for the uplift of circular shallow footings.

Mariupol'skii (13) proposed that the failure surface for very shallow anchors arose not from the sliding of one part over another but purely from separation along a curved failure boundary. For deep anchors, he developed a theory based on the expansion of cavities, which Vesic (31) also developed with a more rigorous analysis. Vesic further derived a solution based on the expansion of cavities for shallow strip and circular anchors close to the surface of a semi-infinite rigid-plastic solid.

Rowe and Davis (24) developed a finite element approach for predicting the load-displacement behavior of strip anchors pulled vertically or horizontally through an elastic-plastic soil. In particular, they concluded that: (1) Soil dilatancy could significantly increase the ultimate capacity of anchors; (2) for values of  $K_0$  in the range 0.4–1.0 the effect on the collapse load could be neglected; and (3) anchor plate roughness had a negligible effect on the capacity of anchors pulled vertically.

At the present time there is therefore no unanimity of view concerning the geometry or mechanism governing the vertical pull-out capacity of embedded anchor plates in sand. In this paper an attempt is made to describe experimental results in terms of upper and lower bound solutions based on equilibrium and limit analysis approaches.

#### LABORATORY TEST PROGRAM

A program of uplift tests was carried out on rectangular mild steel plates with length/breadth ratios ( $L/B$ ) between 1 and 10 with  $B = 2$  in. (50.8 mm), and on circular plates with diameters ( $D$ ) between 2 in. (50.8 mm) and 3.5 in. (88.9 mm). All the anchor plates used were 0.25 in. (6.35 mm) thick and loaded centrally through a 5/16-in. (8.0-mm) diameter tie rod, except for the rectangular plate of  $L/B = 10$ , which had two tie rods connected at the third points and was stiffened by a second smaller plate secured behind the first. All the anchor plates tested may be regarded as rigid.

Two different test arrangements were used in the laboratory program: for tests in very dense sand, a wooden box of width 4 ft (1.22 m) and length 4 ft, 2 in. (1.28 m) was adopted; and for tests in medium dense sand a concrete cylinder of 3 ft, 3 in. (1.0 m) diameter was used. In both series of tests a constant pull-out rate of approximately 0.028 in./min (0.72 mm/min) was employed with loads and displacements continuously monitored by load cells and displacement transducers.

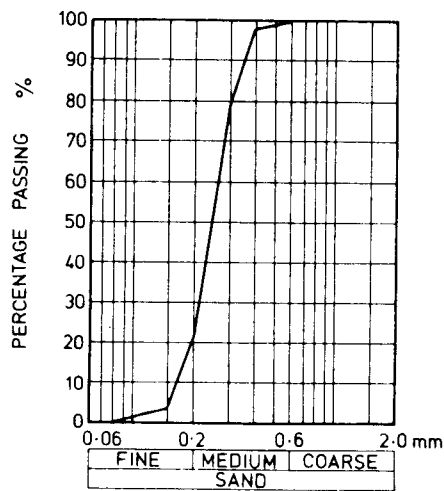


FIG. 1.—Grading Curve of Sand

TABLE 1.—Properties of Sand

State (1)	Density (mg/m <sup>3</sup> ) (2)	Relative density (%) (3)	Porosity (4)	Angle of shearing resistance, $\phi$ (degrees) (5)	Angle of Friction (Degrees)	
					Sand against polished steel surface, $\delta_p$ (6)	Sand against rough sanded surface, $\delta_r$ (7)
Dense state	1.68 $\pm$ 0.02	85.9	0.37	44	11	42
Medium-dense state	1.56 $\pm$ 0.03	42.1	0.41	36	11	—

The air-dried sand used in the anchor uplift tests was predominantly medium grained, as shown in Fig. 1. To obtain a very dense state, the sand was compacted in layers of 2 in. (51 mm) thickness, and to obtain a medium dense state, the sand was poured from a hopper. The angle of shearing resistance of the sand in both conditions was measured in a direct shear box. The properties of the sand are present in Table 1 together with the angles of friction of the sand against a polished steel plate ( $\delta_p$ ) and against a roughened surface ( $\delta_r$ ). The roughened surface consisted of a coating of sand stuck to a steel plate. The uplift tests were performed with polished steel plates, except those shown in Fig. 3(b), which used a plate with a roughened upper surface.

#### LABORATORY TEST RESULTS

The results of the uplift tests on anchor plates embedded in very dense sand are shown as dimensionless plots in Figs. 3 and 5. The ultimate uplift resistance ( $P$ ) and corresponding displacement were defined as occurring at the point where the loading attained a peak value, or where the displacements began to take place under essentially constant load. Typical load/displacement plots for tests in very dense sand are shown in Fig. 2(a).

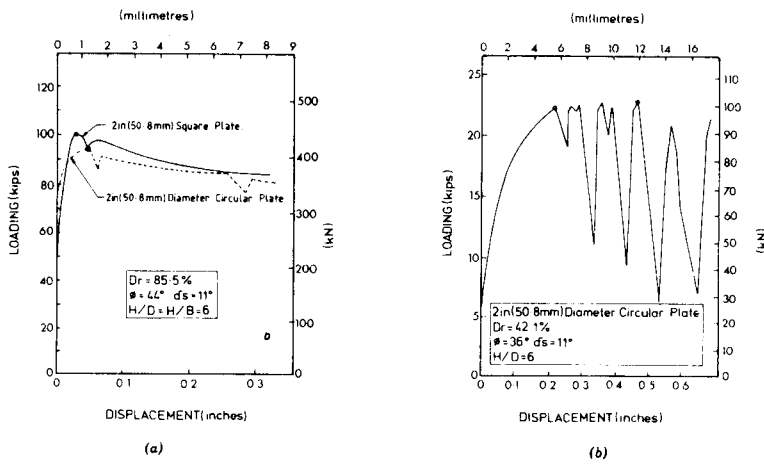


FIG. 2.—Typical Load/Displacement Curves: (a) Very Dense Sand; (b) Medium Dense Sand

In the medium dense sand tests, a typical load-displacement plot from which is shown in Fig. 2(b), the behavior was more erratic, particularly in the tests with circular plates. The initial failure values at which the first sharp drop in uplift loading occurred and the absolute peak loading values have been plotted in Figs. 4 and 6. The erratic behavior resembles that which might be expected in loose sand.

It was found that repeated tests in both very dense and medium dense sand yielded values of ultimate uplift resistance ( $P$ ) within  $\pm 2.5\%$  of the mean. The displacements at ultimate loading showed slightly greater variation.

Several further observations and conclusions from the uplift tests are outlined as follows:

1. As shown in Fig. 3(a), which presents the results of uplift tests on rectangular anchor plates in very dense sand, the dimensionless load coefficient  $P/\gamma AH$  and the corresponding displacement at failure increase with increase of  $H/B$  ratio and decrease of  $L/B$  ratio (where  $A$  is the cross-sectional area of the anchor plate,  $H$  is the depth of embedment, and  $\gamma$  is the unit weight of the sand). The experimental results suggest that the behavior of the plate of  $L/B = 10$  approximates closely to that of a true strip anchor ( $L = \infty$ ) and does not greatly differ from that of a plate of  $L/B = 5$ , at least for small  $H/B$  ratios. The effect of  $L/B$  on the uplift resistance of rectangular shape anchor plates may be conveniently expressed as a dimensionless shape factor defined as

$$\text{Shape Factor} = \frac{\text{load per unit length of the rectangular anchor}}{\text{load per unit length of a strip anchor } (L/B \geq 10)} \dots \dots (1)$$

Fig. 3(c) shows a plot of experimental shape factor against  $H/B$  ratio.

2. As shown in Fig. 3(b), increase of the surface friction angle from  $\delta_s = 11^\circ$  to  $\delta_s = 42^\circ$  increased the ultimate uplift loadings and also increased

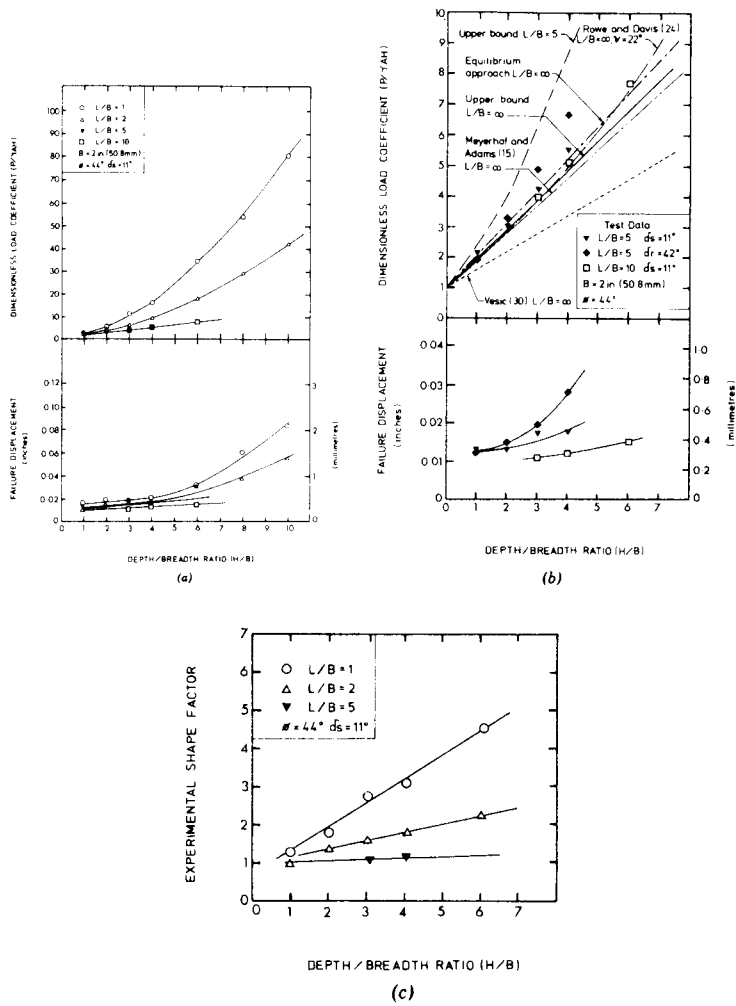


FIG. 3.—(a) Rectangular Anchor Plates in Very Dense Sand; (b) Comparison of Experimental Results and Theoretical Solutions for Rectangular Anchor Plates in Very Dense Sand; (c) Experimental Shape Factors

the corresponding displacements for a plate of  $L/B = 5$ .

3. As might be expected, while the dimensionless load coefficient  $P/\gamma AH$  is greater in very dense sand than in medium dense sand, the corresponding displacements are considerably less.

4. For circular anchor plates in dense sand, as shown in Fig. 5, it appears reasonable to conclude that a consistent relationship exists for both the dimensionless load coefficients  $P/\gamma AH$  and the corresponding displacements for all plates tested when plotted against  $H/D$ . Similar relationships do not appear to exist for the medium dense sand tests shown in Fig. 6.

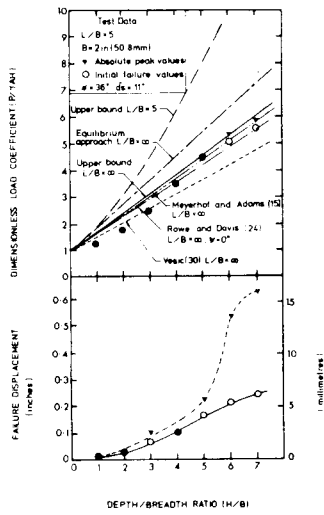


FIG. 4.—Comparison of Experimental Results and Theoretical Solutions for Rectangular Anchor Plates in Medium Dense Sand

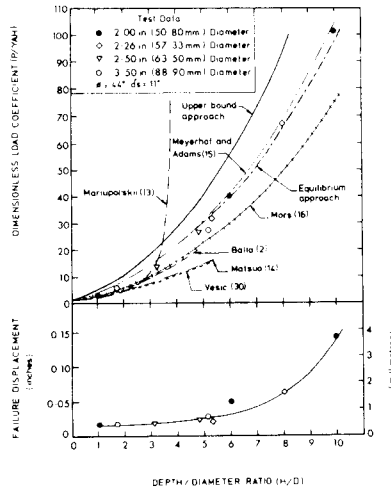


FIG. 5.—Comparison of Experimental Results and Theoretical Solutions for Circular Anchor Plates in Very Dense Sand

5. The results for vertical uplift in very dense sand suggest that values of  $P/\gamma AH$  for a circular plate are on average approximately 1.26 times those for a square plate for  $H/B = H/D$ .

#### ANALYSIS OF STRIP ANCHORS

**Equilibrium Approach.**—Fig. 7(a-b) gives details of an equilibrium analysis for the vertical uplift of a strip anchor. In analyzing the uplift problem, the equilibrium of the wedge  $abc$  is considered. The ultimate uplift resistance is then given by the weight of soil vertically above the anchor plate ( $W_{hd}$ ) added to twice the sum of the weight of soil contained in wedge  $abc$  ( $W_{w1}$ ) and the vertical component of the frictional resistance force ( $F$ ). The ultimate uplift resistance may be written in dimensionless form as follows (for a strip anchor,  $P$  = the ultimate uplift resistance per unit length of plate):

$$\frac{P}{\gamma BH} = 1 + \frac{H}{B} \frac{\tan \theta \tan \delta_w}{\tan \delta_w - \tan (\phi - \alpha)} \dots \dots \dots (2)$$

where  $\phi \geq \delta_w \geq (\phi - \alpha) \geq 0$ .

If the equality  $\alpha = \theta$  is adopted (strictly, this requires a straight-line failure configuration), the analysis is simplified and  $\phi \geq \theta \geq 0$ . Thus limits can be placed on the uplift capacity of a strip anchor.

$$1 \leq \frac{P}{\gamma BH} \leq 1 + \frac{H}{B} \tan \phi \dots \dots \dots (3)$$

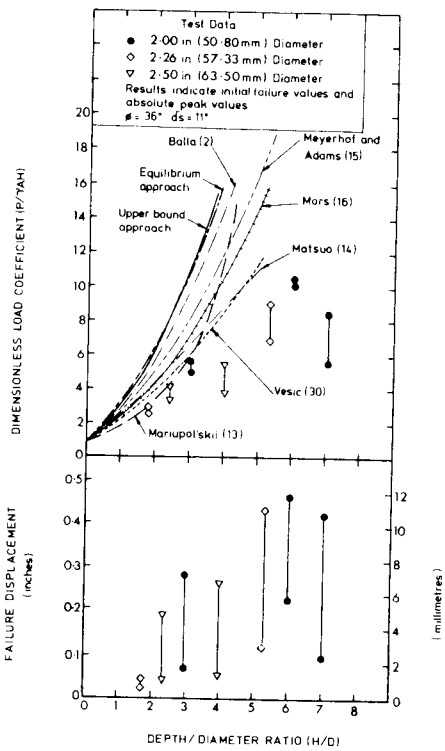


FIG. 6.—Comparison of Experimental Results and Theoretical Solutions for Circular Anchor Plates in Medium Dense Sand

In Eq. 3 the lower limit to the ultimate uplift resistance is obviously unrealistic and implies that it is equal to the weight of soil vertically above the anchor plate.

A practical approach would be to take an average value of  $\theta = \alpha = \phi/2$  and a mid-range value of  $\delta_{cr} = 3\phi/4$ .

Eq. 2 may then be rewritten

$$\frac{P}{\gamma BH} = 1 + \frac{H}{B} \left( \sin \phi + \sin \frac{\phi}{2} \right) \dots \dots \dots (4)$$

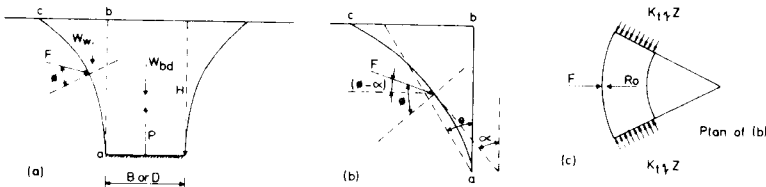


FIG. 7.—Definition of Parameters in Equilibrium Analysis

This will be termed the equilibrium formulation for a strip anchor.

**Limit Analysis Approach.**—An alternative method of estimating the ultimate uplift resistance of a strip anchor is to adopt a limit analysis approach based on the theory of plasticity (5). A possible lower bound solution in which the yield condition is nowhere violated and equilibrium is maintained throughout the soil mass is shown in Fig. 8(a). This gives the uplift resistance as equal to the lower limit of Eq. 3. Although this is a poor lower bound solution, there is difficulty in determining an effective means of altering the stress field to obtain a more satisfactory solution.

In considering an upper bound solution, it should be noted that for a cohesionless medium no energy is dissipated on any displacement discontinuity. This follows from the fact that for a material obeying an associated flow rule the displacement vector is perpendicular to the frictional resistance force on this boundary. For an upper bound solution it is necessary to find the minimum value of loading computed for any prescribed failure mechanism, and then to examine different mechanisms in a similar way to evaluate as close a bound as possible to the ultimate uplift resistance in the idealized material.

Fig. 8(b) shows a possible straight-line failure mechanism. However, in the limit, failure mechanisms consisting of any form and number of straight line discontinuities that are compatible with a displacement of the anchor reduce to a single final solution. For the general failure configuration shown in Fig. 8(b), equating the work done by external forces (including the soil's weight) to the dissipation of energy, which is zero since the soil is cohesionless, the following is obtained:

$$\frac{P}{\gamma BH} = 1 + \frac{H \tan(\phi + w) \tan(\beta - w) - \tan w \tan(\beta - w - \phi)}{B \tan(\phi + w) + \tan(\beta - w - \phi)} \dots \dots \dots (5)$$

The larger the value of  $\beta$  the greater  $P/\gamma BH$ . Since a minimum solution is sought, the smallest possible value of  $\beta$  must be found. Since  $(\beta - w - \phi) \geq 0$ , the minimum value is given by  $\beta = w + \phi$  and Eq. 4 reduces to the upper limit of Eq. 3, i.e.

$$\frac{P}{\gamma BH} = 1 + \frac{H}{B} \tan \phi \dots \dots \dots (6)$$

This will be termed the upper bound solution. The failure boundaries

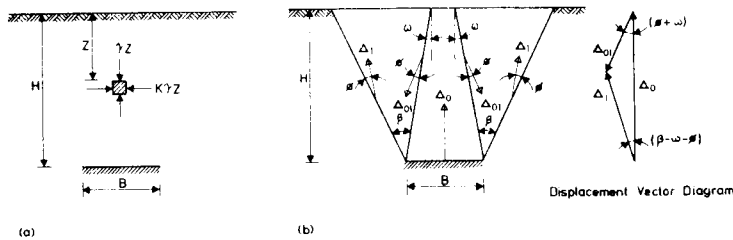


FIG. 8.—Limit Analysis Solutions: (a) Lower Bound Solution; (b) Upper Bound Failure Mechanism



consist of two straight lines inclined outwards at an angle  $\phi$  to the vertical at the plate edges. This solution is considered to yield a minimum upper bound for the problem because it is difficult to imagine any other suitable failure mechanism that leads to a lower ultimate uplift resistance.

**Comparison of Experimental and Theoretical Results for Strip Anchors.**—Figs. 3(b) and 4 compare the equilibrium and upper bound solutions for a strip anchor ( $L = \infty$ ) with the experimental results for plates with  $L/B = 5$  and  $L/B = 10$ . Also shown are the predicted results for strip anchors from the theories of Vesic (31), Meyerhof and Adams (16), and Rowe and Davis (24).

Vesic theory is shown to underpredict the experimental results in very dense sand, but it provides reasonable correlation with the experimental evidence in medium dense sand. The theory of Meyerhof and Adams, on the other hand, is shown to yield reasonable results in both very dense and medium dense sand for polished plates ( $\delta_s = 10.6^\circ$ ), and to give values only slightly smaller than those of the upper bound approach with a plate of  $L/B = \infty$ . The equilibrium approach yields close agreement with the experimental results for polished plates in very dense sand but overpredicts the results in medium dense sand.

The approach of Rowe and Davis (24) yields a range of results dependent on the assessed dilatancy of the soil. There is reasonable agreement between their theoretical results and the experimental data for polished plates in very dense sand [Fig. 3(b)] if an angle of dilation of  $\psi = 22^\circ$  is adopted in the analysis. However, Rowe and Davis concluded that the effect of plate roughness on anchors pulled vertically could be neglected, which is in contradiction to the limited experimental data for roughened plates in very dense sand. For the results of pull-out tests in medium dense sand presented in Fig. 4, Rowe and Davis's approach shows reasonable agreement with the experimental evidence if it is assumed that  $\psi = 0^\circ$ .

The fact that the upper bound approach for a strip anchor agrees well with the experimental results for polished steel plates is somewhat surprising, since the upper bound for the passive case of loading yields an overestimate for a material obeying an associated flow rule. Thus, it should also yield an overestimate for a material obeying a nonassociated flow rule. (The behavior of real soils is probably more accurately described by a nonassociated flow law where the angle of dilation is less than the angle of shearing resistance.)

The experimental results of Fig. 3(b) indicate that an increase in surface roughness produces an increase in the ultimate uplift resistance. Drucker (4) shows that for an assemblage of elastic-plastic bodies with frictional interfaces "any set of loads which produces collapse for the condition of no relative motion at the interfaces will also produce collapse for the case of finite friction." Thus the upper bound solution for a strip anchor (Eq. 6) that allows no relative motion at the sand-plate interface should overpredict the results for a plate with a frictional interface. Since the vertical uplift problem is symmetrical, it is difficult to imagine a failure mechanism that allows relative motion to occur on the sand-plate interface and yields a lower ultimate uplift resistance than that given by Eq. 6.

**Equilibrium Approach.**—Fig. 7(a-c) is appropriate for circular plate anchors. The analysis is similar to that for a strip anchor, except that for the circular anchor case the analysis is in terms of total forces rather than loads per unit length. The ultimate uplift resistance may be written in dimensionless form as

$$\frac{P}{\gamma AH} = 1 + \frac{2H}{D} \frac{\tan \theta \tan \delta_{ic}}{\tan \delta_{ic} - \tan (\phi - \alpha)} \left\{ 1 + \frac{2H}{3D} [\tan \theta + K_t \tan (\phi - \alpha)] \right\} \quad (7)$$

where  $\phi \geq \delta_{ic} \geq (\phi - \alpha) \geq 0$ .

If the equality  $\theta = \alpha$  is adopted, as for the case of a strip anchor, then Eq. 7 is simplified and  $\phi \geq \theta \geq 0$ . Thus limits can be placed on the uplift of a circular anchor:

$$1 \leq \frac{P}{\gamma AH} \leq 1 + 2 \frac{H}{D} \tan \phi \left( 1 + \frac{2H}{3B} \tan \phi \right) \dots \dots \dots (8)$$

The lower limit to the ultimate uplift resistance is obviously unrealistic since it implies that it is merely equal to the weight of soil vertically above the anchor plate.

A practical approach would be to adopt Eq. 7 and take an average value of  $\theta = \alpha = \phi/2$  and a mid-range value of  $\delta_{ic} = 3\phi/4$  as for a strip anchor. It only remains to assign a value to  $K_t$  to obtain a solution. As an approximation,  $K_t$  can be taken equal to  $K_0 = 1 - \sin \phi$ , where  $K_0$  is the lateral earth pressure coefficient at rest. Thus

$$\frac{P}{\gamma AH} = 1 + 2 \frac{H}{D} \left( \sin \phi + \sin \frac{\phi}{2} \right) \left( 1 + \frac{2H}{3D} \tan \frac{\phi}{2} (2 - \sin \phi) \right) \dots \dots \dots (9)$$

This will be termed the equilibrium formulation for a circular anchor.

**Limit Analysis Approach.**—For the limit analysis of circular anchors, conclusions similar to those for strip anchors can be drawn. A poor lower bound solution is given by the lower limit of Eq. 8. The failure mechanism of the minimum upper bound solution appears to be a straight line inclined at an angle  $\phi$  to the vertical at the plate edge. It may be shown that this upper bound solution is equal to the upper limit in Eq. 8.

**Comparison of Experimental and Theoretical Results for Circular Anchors.**—Figs. 5 and 6 compare the experimental results for polished circular anchor plates embedded in very dense and medium dense sand with the equilibrium and upper bound analyses developed in this paper and some alternative theories. The equilibrium approach, although yielding excellent agreement with the experimental results in very dense sand, yields values of ultimate uplift resistance greater than the experimental values in medium dense sand. The upper bound approach, on the other hand, yields greater values than the experimental results in both very dense and medium dense sands. The semi-empirical analysis of Meyerhof and Adams (16), which shows good agreement with the experimental results in dense sand and agrees closely with the equilibrium approach, also overpredicts the experimental results in less dense sand.

Similar to the conclusion of Sutherland (27), comparison of the experimental results with Balla's theory (2) suggests that the latter theory is too insensitive to the variation of  $\phi$ , because it yields values for ultimate resistance less than the experimental results in very dense sand and greater than the experimental results in medium dense sand. Similar conclusions can be drawn about the theories of Vesic (31), Mors, (16), and Matsuo (14). The theories of both Vesic and Matsuo do, however, show better correlation with experimental evidence in medium dense sand than most of the other theories considered.

The ultimate uplift resistance predicted by Mariupol'skii's (13) method is also shown in Figs. 5 and 6. It is obvious why the theory is limited to very shallow depths, for the curve quickly rises to infinite values. It is suggested that this method is too unreliable for practical purposes.

**ANALYSIS OF RECTANGULAR ANCHORS**

**Limit Analysis Approach.**—Following the conclusions of the previous sections, the most appropriate failure configuration for a rectangular anchor plate appears to consist of a straight-line failure plane inclined at an angle  $\phi$  to the vertical at the edge of the plate. At the corners the configuration consists of a portion of a circular cone. The resulting uplift equation is given as follows:

$$\frac{P}{\gamma AH} = 1 + \frac{H}{B} \tan \phi \left( 1 + \frac{B}{L} + \frac{\pi H}{3L} \tan \phi \right) \dots \dots \dots (10)$$

**Comparison of Experimental and Theoretical Results for Rectangular Anchor Plates.**—Comparison of the values predicted by Eq. 10 with the experimental results for rectangular anchor plates indicates an overestimate of the uplift resistance. The results, however, show a better correlation with the experimental evidence than the method proposed by Meyerhof and Adams (16). Figs. 3(b) and 4 show a plot of the theoretical values from Eq. 10 for a rectangular plate of  $L/B = 5$ . It may be noted that for increase of the interface friction angle, the experimental results tend to approach the theoretical results for a plate of  $L/B = 5$ . As pointed out previously for the case of strip anchors, the upper bound approach should overpredict the results for a plate with a perfectly frictional interface ( $\delta_c = \phi$ ). Thus it is tentatively suggested that the upper bound for a rectangular anchor plate will show more acceptable correlation with the results for a plate with a perfectly frictional interface.

**SUMMARY AND CONCLUSIONS**

The work described within this paper is part of a study made of the passive resistance of anchorages in sand (16). The results of laboratory tests are reported, and comparisons are made with previously published theoretical solutions and equilibrium and limit analysis solutions as developed within this paper.

The main conclusions of the experimental work were as follows:

1. For the uplift of rectangular plates in very dense sand, the dimensionless load coefficient  $P/AH$  and the corresponding displacement at

failure increase with an increase of  $H/B$  ratio and a decrease of  $L/B$  ratio. The dependence of uplift resistance on  $L/B$  is described in terms of a shape factor. There is also a marked increase in  $P/\gamma AH$  and the corresponding displacement in very dense sand for plates with high surface friction angle compared to plates with polished surfaces.

2. Significant differences in behavior were noted between plates embedded in very dense sand and those embedded in medium dense sand. While the dimensionless load coefficient  $P/\gamma AH$  is greater in very dense sand, the corresponding displacements are considerably less. Of particular concern is the recorded behavior of circular plates in medium dense sand where large abrupt decreases in uplift resistance were recorded prior to the absolute maximum uplift resistance.

3. For circular plates in very dense sand, there appears to be a consistent relationship for both the dimensionless load coefficients  $P/\gamma AH$  and the corresponding displacements, for all plates tested, when plotted against  $H/D$ . Similar relationships do not appear to exist in medium dense sand. In very dense sand, the  $P/\gamma AH$  values for circular plates are, on average, approximately 1.26 times those of square plates for  $H/B = H/D$ .

The theoretical solutions for strip and circular anchors examined show a scatter of plots, in some cases showing very poor correlation with the experimental results. These discrepancies can probably be attributed to some extent to an inability to describe mathematically the stress history or degree of overconsolidation of the sand arising from the sand placement techniques (7). No solution examined can be said to give good agreement with all the experimental results, particularly for circular plates in medium dense sand where all theoretical solutions overpredict the ultimate uplift resistance. The upper bound and equilibrium solutions developed in this paper correlate as favorably with the experimental evidence as the other theories examined, although they tend to overpredict rather than underpredict the uplift resistance.

The upper bound solutions should overpredict the results for a plate with a perfectly frictional interface ( $\delta_r = \phi$ ). It is therefore tentatively suggested that better correlation between upper bound solutions and experimental results would be obtained if plates with roughened surfaces rather than polished surfaces were tested.

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## APPENDIX II.—NOTATION

The following symbols are used in this paper:

- $A$  = area of face of anchor plate;
- $B$  = breadth of rectangular anchor plate;
- $D$  = diameter of circular anchor plate;
- $H$  = depth of embedment;
- $K, K_t$  = coefficient of lateral earth pressure;
- $K_0$  = coefficient of earth pressure at rest;
- $L$  = length of rectangular anchor plate;
- $P$  = ultimate uplift resistance; ultimate uplift resistance/unit length for strip anchor ( $L = \infty$ );
- $R_0$  = magnitude of resultant radial force due to circumferential pressure  $K_t\gamma Z$  in Fig. 7(c);
- $W_{bd}$  = weight of soil vertically above anchor plate in Fig. 7(a);
- $W_w$  = weight of soil contained in wedge  $abc$  in Fig. 7(a);
- $Z$  = depth below soil surface;
- $\alpha$  = inclination to the vertical of the curved failure surface  $ac$  in Fig. 7(b) at the point where the resultant frictional force  $F$  acts at the angle of shearing resistance  $\phi$  to the vertical;
- $\gamma$  = unit weight of soil;
- $\Delta$  = displacement;
- $\delta_s, \delta_r$  = friction angles, see Table 1;
- $\delta_w$  = inclination of the resultant of forces  $W_w$  and  $F$  to the horizontal in Fig. 7(a);
- $\theta$  = angle of the equivalent weight line from the vertical for wedge  $abc$  in Fig. 7(b);
- $\phi$  = angle of shearing resistance; and
- $\psi$  = angle of dilation.