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Resistance of passive inclined anchors in cohesionless medium

E. J. MURRAY* and J. D. GEDDES†

The passive resistance of inclined anchor plates in sand is examined. Laboratory experimental results are presented for the ultimate passive resistance, and the corresponding displacements, of rectangular anchor plates pulled at angles of inclination between the vertical and the horizontal through very dense sand. The results of ultimate passive resistance are compared with theoretical solutions based on the upper and lower bound limit theorems of soil plasticity. A general lower bound solution for a strip anchor pulled at various angles of loading cannot be readily formulated, and only a poor lower bound solution for the case of horizontal loading is determined. However, it is shown that provided interface friction is taken into account, upper bound solutions for a strip anchor compare favourably with the experimental evidence. Good agreement is shown between the upper bound solutions and the equivalent free surface stress characteristic solution, and experimental data, of Neely *et al.* (1973) for the horizontal translation of anchor plates in medium dense sand. Comparison is also made between the experimental results of this Paper and the finite element approach of Rowe & Davis (1982) for anchors pulled horizontally through an assumed elastic–plastic soil.

KEYWORDS: **anchors; failure; model tests; plasticity; sands; soil–structure interaction.**

L'article examine la résistance passive des plaques d'ancrage inclinées. Les résultats de quelques expériences effectuées en laboratoire sont présentés concernant la résistance passive limite et les déplacements correspondants de plaques d'ancrage rectangulaires arrachées selon des directions comprises entre la verticale et l'horizontale à travers du sable de grande densité. Les résultats pour la résistance passive limite sont comparés avec des solutions théoriques basées sur les valeurs supérieures et inférieures possibles des théories de la plasticité du sol. Il est difficile de formuler une solution générale pour les valeurs inférieures pour une plaque d'ancrage extraite à des angles différents tandis qu'on ne détermine qu'une solution peu satisfaisante pour les valeurs inférieures dans le cas du chargement horizontal. Néanmoins, à condition qu'on tienne compte du frottement d'interface, les solutions dans les valeurs supérieures s'accordent très bien avec les résultats expérimentaux. On démontre un accord satisfaisant entre les solutions limites des valeurs supérieures et la solution equivalente basée sur les caractéristiques de contrainte superficielle libre et les données expérimentales de Neely et autres (1973) pour la translation horizontale des plaques d'ancrage dans du sable de densité moyenne. On compare aussi les résultats décrits dans cet article avec la méthode aux éléments finis de Rowe et Davis (1982) pour des ancrages horizontalement à travers un sol élastoplastique.

NOTATION

- A area of anchor plate
- B breadth of anchor plate
- c unit cohesion
- D dimensionless parameter (Appendix 1 equation (7))
- H depth of embedment of anchor plate (to bottom of plate)
- K_0 coefficient of earth pressure at rest
- K_p coefficient of passive resistance
- L length of anchor plate
- m degree of shear mobilization, Neely *et al.* (1973), (Fig. 12)
- P ultimate passive resistance for rectangular anchors and per unit length for strip anchors
- W_1, W_2, W_3 work done
- z depth
- α angle (Appendix 1 equations (6) and (7))
- δ interface friction angle
- $\dot{\varepsilon}_1^p$, $\dot{\varepsilon}_3^p$ major and minor principal plastic strain increments (Fig. 7)

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$\dot{\varepsilon}^p$	plastic normal strain increment (Fig. 7)
γ	unit weight of soil
$\dot{\gamma}^p$	plastic shear strain increment (Fig. 7)
Δ, Δ_1, Δ_2, etc.	displacements and relative displacements
σ_1, σ_3	major and minor principal stresses
σ_n	normal stress
τ_n	shearing stress
ϕ	angle of shearing resistance
ψ	angle of dilation
ω	angle of inclination of anchor tie rod from the vertical
ν, ρ	angles (Fig. 9)
Ω, λ	angles (Fig. 10)

INTRODUCTION

In practice there are a number of anchor types which rely on both passive resistance and skin friction to resist inclined tension loads arising from structural loading. This study is concerned with the passive resistance of strip and rectangular anchors embedded in sand and pulled at angles between the vertical and the horizontal. Most previous work has been concerned with either the vertical uplift problem or with anchors pulled horizontally. Murray & Geddes (1987) have presented a study of the vertical uplift of anchors, and Dickin & Leung (1983; 1985) have reviewed the problem of strip and rectangular anchors pulled horizontally. There is far less information available on inclined anchors. The literature on inclined passive anchors of various shapes includes papers by Kananyan (1966), Larnach (1972; 1973), Harvey & Burley (1973), Meyerhof (1973), Colp & Herbich (1975), Das & Seely (1975) and Larnach & McMullan (1975).

Mathematically the simplest form of passive plate anchor consists of a rigid infinite strip which may be treated as a plane problem; this then leads to a consideration of plates of finite length which must be treated as three-dimensional problems. Bearing this in mind, a series of small-scale laboratory pulling tests on anchors inclined between the vertical and the horizontal was carried out. The results of these tests are presented. Such testing allows a close control of the variables encountered in practice and, although the results do not provide a direct comparison with full-scale behaviour (Dickin & Leung, 1983; 1985), they can be of value in providing an understanding of behaviour patterns and give a guide to full-scale performance, particularly when examined in conjunction with developed theoretical solutions.

The laboratory test results are compared with theoretical solutions based on the lower and upper bound limit theorems of soil plasticity (Chen, 1975; Davis, 1968; Drucker, 1954; Drucker *et al.*, 1952; Koiters, 1960). A simple statically admissible stress field solution is examined for the lower bound solution of a strip anchor pulled horizontally, and displacement collapse mechanisms are examined for the minimum upper bound solution of a strip anchor pulled at various angles of loading. The limit theorems rely on the normality principle which requires that the angle of dilation ψ equals the angle of shearing resistance ϕ, and the appropriateness of this assumption will be discussed. The test results and analyses form part of a study on ground anchors by Murray (1977).

An interesting comparison with the upper and lower bound limit analysis solutions is provided by the experimental results and stress characteristic solutions of Neely *et al.* (1973) for horizontally translating strip anchors. Their theoretical solutions are based on the procedures of Sokolovski (1960; 1965) and the stress characteristic configuration is determined assuming two different stress boundary conditions: namely, a surcharge loading and an equivalent free surface loading. The latter form of loading can be considered to possess a varying degree of shear mobilization, thus producing a range of failure loads. These approaches do not conform to either lower or upper bound limit analysis solutions because they do not demonstrate that stress equilibrium is maintained throughout the whole of the soil mass with the yield condition nowhere violated, or that they are kinematically acceptable.

The finite element approach of Rowe & Davis (1982), assuming an elastic–plastic soil, is also examined for the case of horizontally translating strip anchors, and is compared with the experimental evidence of this Paper. In particular they examined the effects of soil dilatancy, initial stress state K_0, roughness of anchor plate and depth of embedment on the load–displacement behaviour.

DETAILS OF LABORATORY TESTING

The results of laboratory pull-out tests on rectangular anchor plates of length/breadth $(L/B) =$ 1, 2, 5 and 10 loaded at angles of inclination to the vertical ω from 0° to 90° and embedded at depth/breadth (H/B) ratios of up to 8 are reported. Polished steel anchor plates with $B = 50{\cdot}8$ mm were used in the tests. With one exception, the thickness of all the anchor plates used was 6·35 mm with each plate loaded centrally through a tie rod, of 8 mm dia. The exception to the above was the plate of $L/B = 10$ which was stiffened by a second plate of slightly smaller dimensions secured behind the first and which was loaded through two 8 mm dia. tie rods connected at the third points. All the plates tested may be regarded

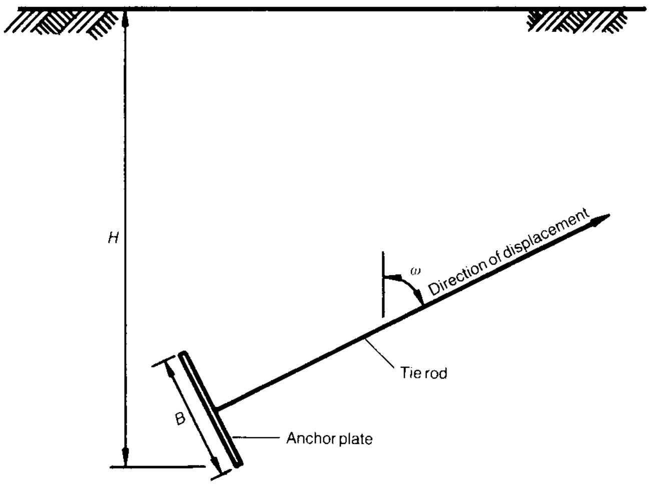

Fig. 1. Geometry of problem

as rigid. The geometrical parameters are shown in Fig. 1.

The grading curve of the sand used in the anchor tests is shown in Fig. 2. The air-dried sand, known as Portishead Fines, is predominantly medium-grained and it was compacted in layers by vibration with the anchor plate fixed in position in the test container. A layer thickness of 50·8 mm was adopted and a density of 1·68 Mg/m^3 ($\pm 0{\cdot}9\%$) achieved. At this density, direct shear box tests on the sand yielded peak strength parameters of $c = 0$, $\phi = 43{\cdot}6^\circ$. Modified shear box tests yielded a peak interface friction angle δ of 10·6° between the sand and a polished steel surface, prepared in a similar manner to the surfaces of the steel anchor plates. The stress range used in the direct shear box tests was that anticipated within the anchor tests.

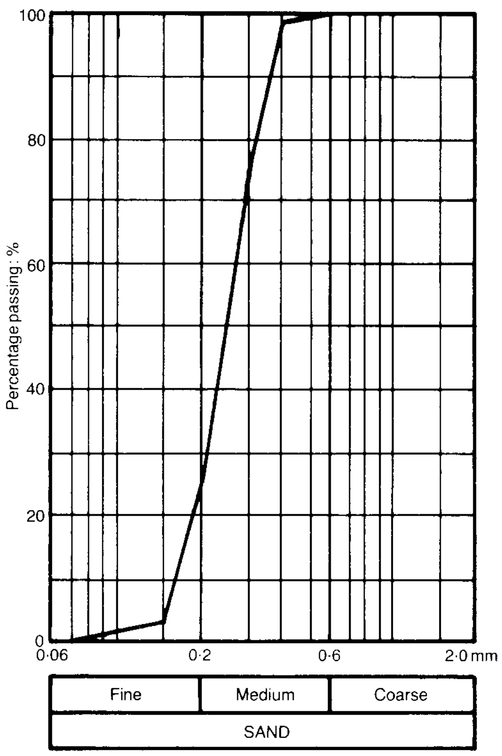

Fig. 2. Grading curve for sand

The test rig is outlined in Fig. 3. A pull-out rate of approximately 0·72 mm/min was used, with loads and displacements continuously monitored by load cells and displacement transducers. A proving-ring and displacement dial gauges were also mounted to give visual indications of anchor behaviour during loading.

LABORATORY TEST RESULTS

In analysing the pull-out results, the ultimate passive resistance P was defined as that occurring at the point where the loading attained a peak value, or where displacements began to take place under essentially constant load. Typical load–displacement plots are shown in Fig. 4.

The results for ultimate passive resistance are presented as dimensionless plots of $P/\gamma AH$ (where A is the area of the anchor plate and γ is the unit weight of the soil) against H/B in Fig. 5a–e. These dimensionless quantities are considered to be the most appropriate for examining the passive anchor problem. The anchor plate displacements at the defined ultimate passive resistance are also presented. The theoretical analysis that follows,

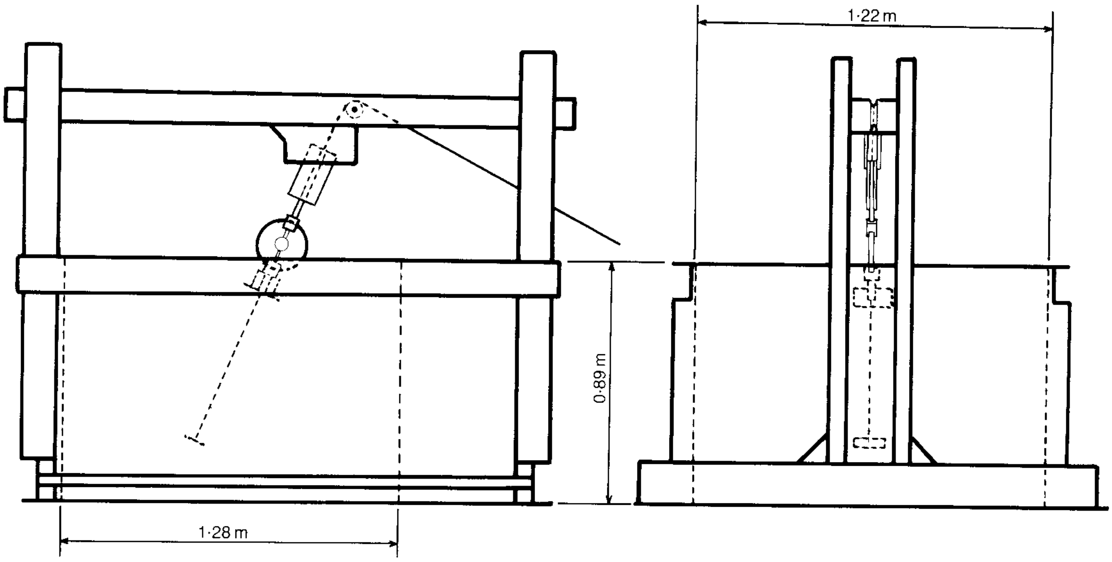

Fig. 3. Details of anchor test rig

although in part based on kinematic considerations, does not produce dimensionless displacement quantities comparable to the dimensionless loading $P/\gamma AH$. For this reason, to give a clear indication of the magnitudes of movements, the displacements are reported directly. Several tests were repeated to assess the reliability of the results obtained. Differences of less than 5% were

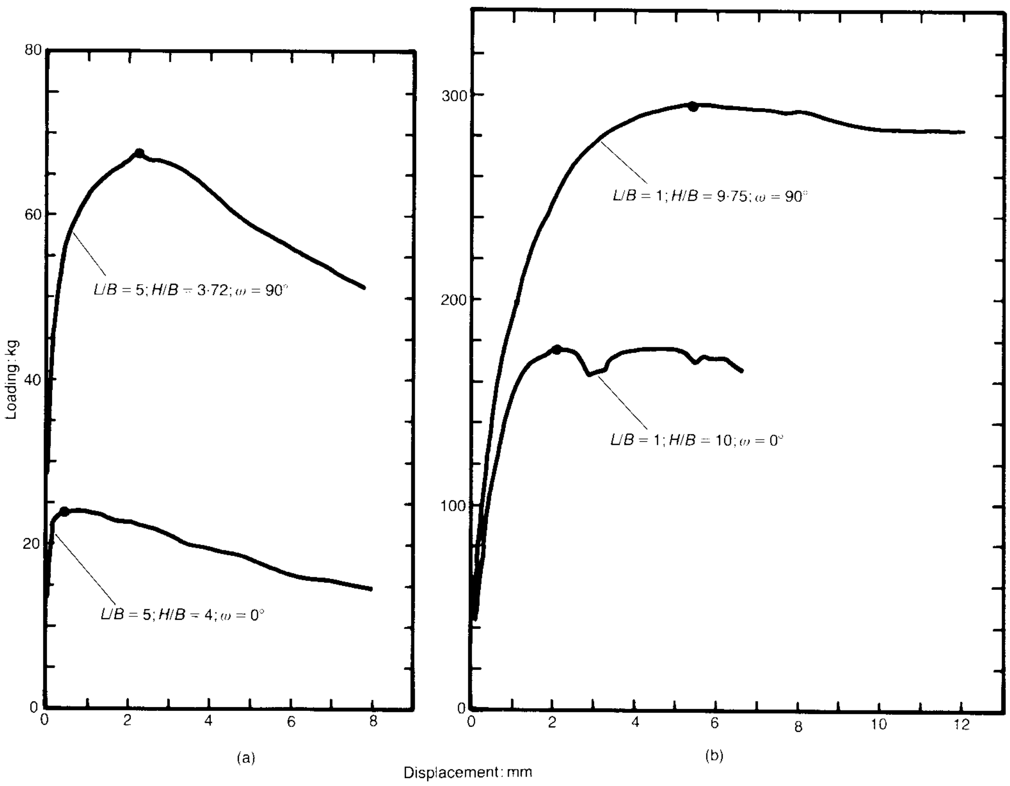

Fig. 4. Load/displacement plots: (a) for plate of $L/B = 5$; (b) plate of $L/B = 1$

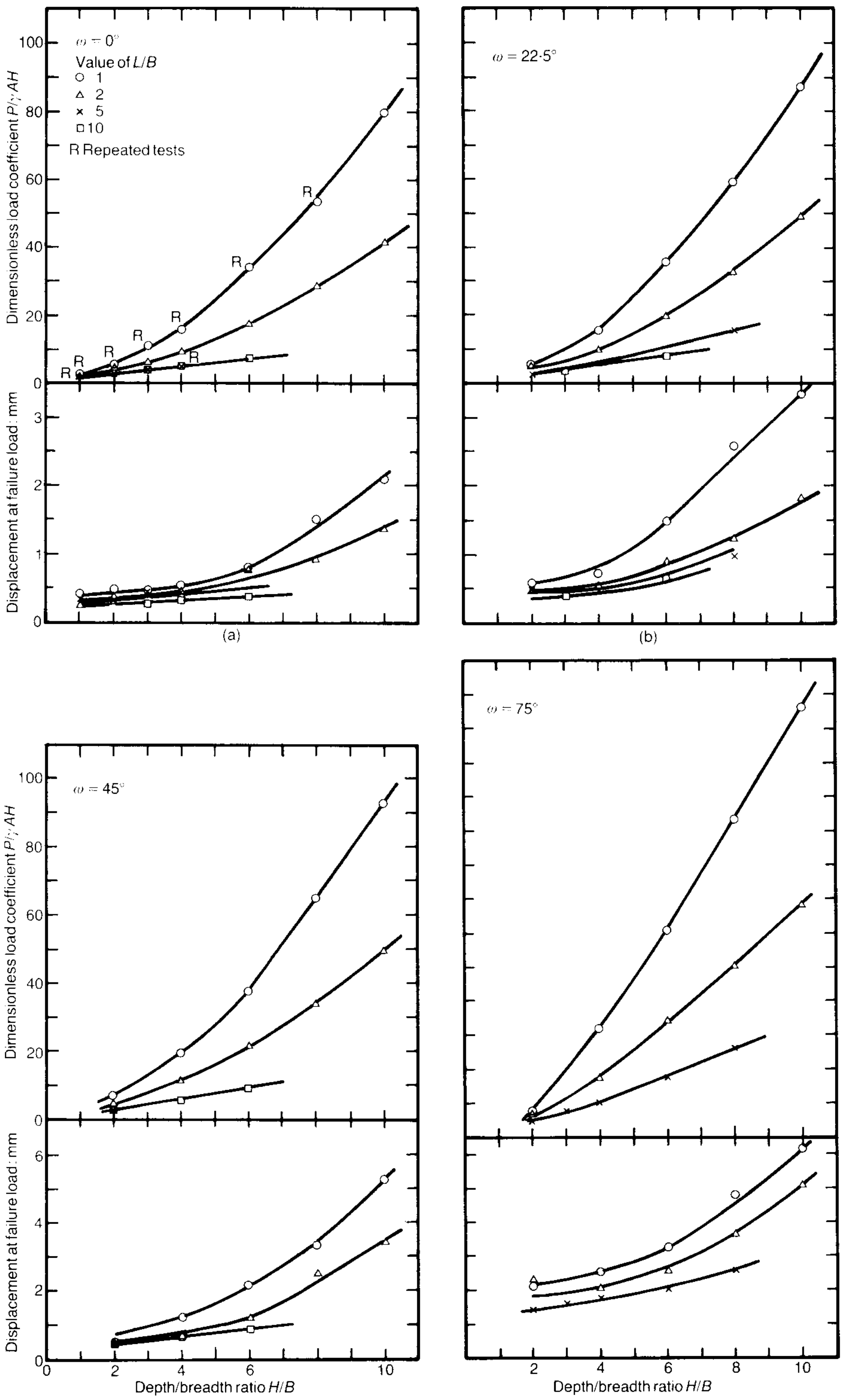

Fig. 5. Failure loading and corresponding displacement: (a) for ω = 0°; (b) ω = 22·5°; (c) ω = 45°; (d) ω = 75°

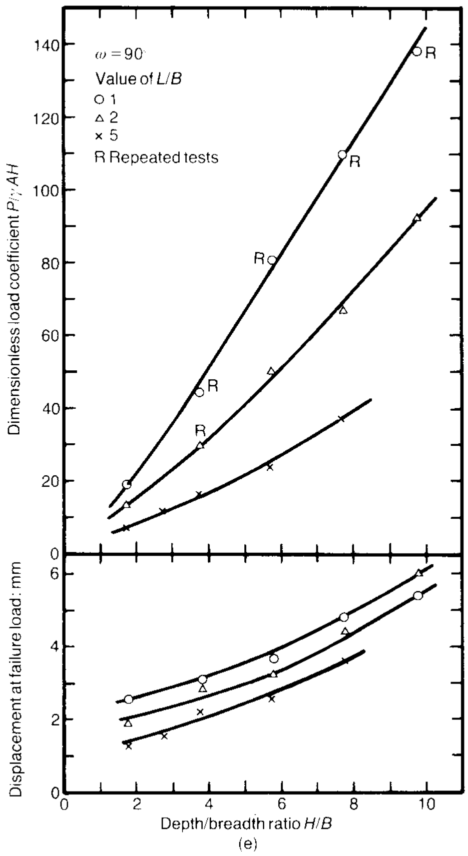

Fig. 5. (continued): (e) for ω = 90°

noted in the values of P, with the corresponding displacements showing only slightly greater variation.

The $P/\gamma AH$ values and corresponding displacements increase with increasing H/B ratio and decrease of L/B ratio. The test results also show that for a given plate, $P/\gamma AH$ and the corresponding displacements increase with the inclination of the direction of pull-out. More than 80% of the increase in $P/\gamma AH$, when ω varies between 0° and 90°, is associated with the interval $\omega = 45°$ to $\omega = 90°$.

In examining the results it is convenient to introduce a shape factor defined by the expression

Shape factor =

$$\frac{\text{load per unit length of rectangular plate}}{\text{load per unit length of plate with } L/B \geqslant 5} \quad (1)$$

This definition of shape factor is based on laboratory test results which showed that the behaviour of a plate of $L/B = 10$ approximates closely to that of a true strip anchor ($L/B = \infty$), and that for practical purposes a plate of $L/B = 5$ can be used as a guide to the behaviour of a strip anchor. The definition is consistent with that adopted by Neely *et al.* (1973).

Plots of shape factor (where a plate of $L/B = 5$ has been used as the reference datum in the denominator of equation (1)) are presented in Fig. 6. It appears that, for practical purposes, the shape factor for a given plate and depth/breadth ratio may be regarded as independent of the direction of pull-out, at least for H/B ratios in excess of about 2.

THEORETICAL ANALYSIS

The limit theorems of soil plasticity have been used to estimate the passive resistance of inclined strip anchors ($L/B = \infty$) in a cohesionless medium. The proofs of the limit theorems, which are outlined in a number of publications (e.g. Chen, 1975; Davis, 1968; Drucker, 1954; Drucker *et al.*, 1952; Koiters, 1960), and are based on the principle of virtual work, are not reproduced here. In plasticity theory the use of the term rate of change applied to stress and strain is common. However, in the following an incremental approach has been adopted, as suggested by Wroth (1973). This is considered more appropriate for the failure loading of a soil.

The upper and lower bound theorems enable the true theoretical failure load of an idealized material to be bracketed and in some cases to be actually defined if there is convergence of the bound solutions. The relevance of such solutions depends on the extent to which the soil can be reasonably represented by the idealized mathematical model.

Although theorems have been developed which yield close bounds to the collapse loading of a number of problems that involve an idealized material which obeys an associated flow rule, only restricted theorems have been formulated for the more general class of materials which deform under non-associated flow conditions. The existence of an associated flow rule for a Mohr–Coulomb ($c - \phi$) type material implies that the angle of dilation ψ is equal to the angle of shearing resistance ϕ. The mathematical advantage of assuming an associated flow law is that the stress and displacement characteristics coincide in the plastic field if it is accepted that the principal axes of stress and strain-increment coincide. This assumption is made in the analyses presented in this Paper. The principle is outlined in Fig. 7.

Lower bound solutions for strip anchor pulled horizontally

A lower bound limit solution requires that equilibrium is maintained throughout the whole soil mass and the yield condition is nowhere

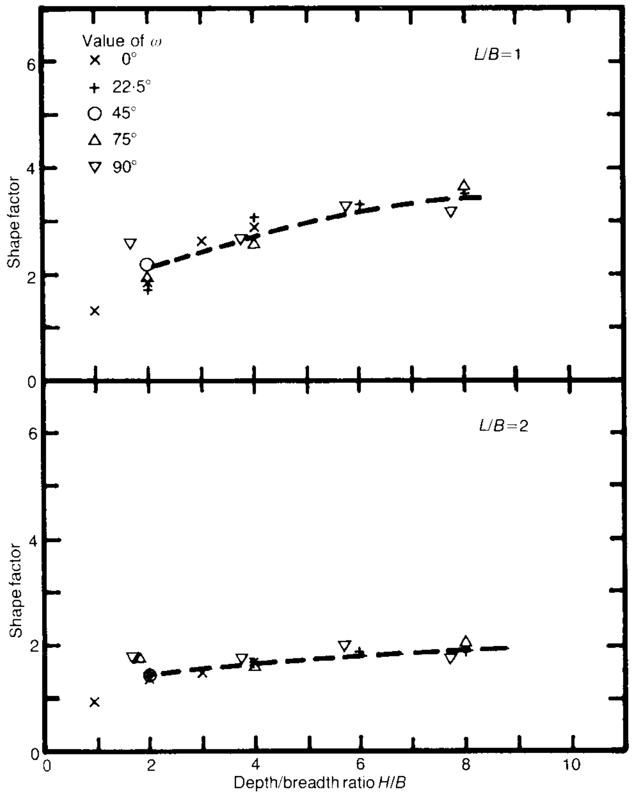

Fig. 6. Shape factor against depth/breadth ratio for rectangular plates of $L/B = 1$ and 2

violated. The lower bound limit solution thus requires the formulation of a statically admissible stress field.

Figure 8 indicates a simple acceptable stress field which when analysed yields the following lower bound solution to the ultimate passive resistance of a strip anchor pulled horizontally

$$\frac{P}{\gamma BH} = K_p(1 - 0{\cdot}5\ B/H) \quad (2)$$

where K_p is the coefficient of passive resistance and P in this instance is the ultimate passive resistance per unit length of anchor plate.

There are difficulties in altering the stress field in order to formulate a general lower bound solution for inclined anchors because interface friction, which gives rise to energy dissipation, is called into play and this invalidates the analysis.

Upper bound solutions for strip anchor

In developing an upper bound solution it is necessary to determine a valid collapse mechanism such that the displacement field throughout is continuous. Two mechanisms of this kind have been examined (Figs 9 and 10) and the collapse loading for each determined by equating the work done by external forces, including the soil's self weight, to the internal dissipation of energy as a result of the soil's inherent shearing resistance. In any region of the mechanisms, the external work done is determined by multiplying the vertical component of displacement by the soil's self weight. Since the soil under consideration is cohesionless, and the idealized material obeys an associated flow rule, the dissipation of internal energy is zero because the displacement vector is perpendicular to the frictional resistance force on any discontinuity.

The interface friction between anchor plate and soil can be taken into account in the upper bound analysis by following the method proposed by Drucker (1954) and allowing dilation on the interface, so that the displacement vector is inclined to this surface at the angle of interface friction δ. This constitutes an upper bound solution for a passive anchor with interface friction.

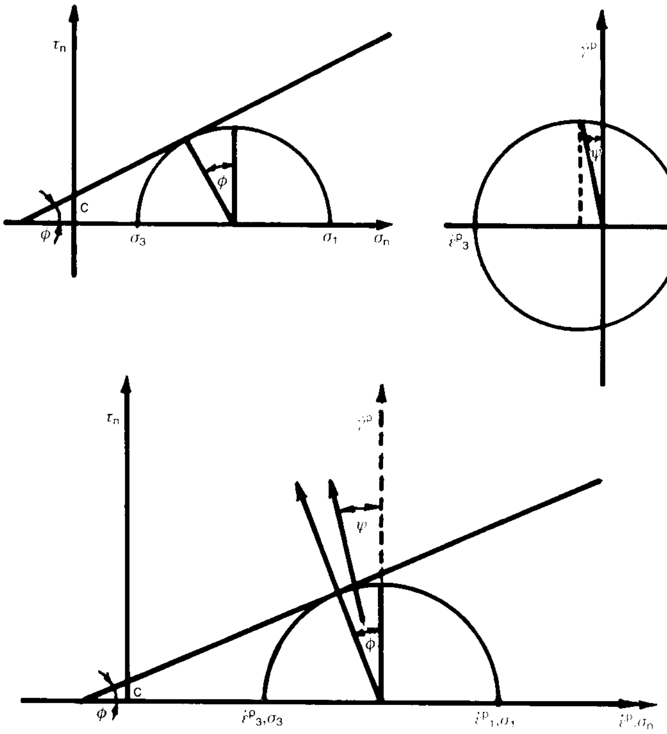

Fig. 7. Definition of angle of shearing resistance and angle of dilation

Thus for mechanism 1, Fig. 9

$$\frac{P}{\gamma BH} = -\frac{(W_1 + W_2 + W_3)}{\gamma BH\Delta_1} \cdot \frac{\cos \delta}{\cos (\rho + \delta)} \quad (3)$$

where W_1, W_2 and W_3 denote the work done against the soil's weight in the three regions of the failure mechanism. Appendix 1 outlines the method for determining the upper bound solution for this mechanism with $\delta > 0$.

For mechanism 2, Fig. 10

$$\frac{P}{\gamma BH} = -\frac{(W_1 + W_2)}{\gamma BH\Delta_1} \cdot \frac{\sin (\lambda + \phi + \omega - \delta)}{\sin^2 (\lambda + \phi + \omega) \cos \delta} \quad (4)$$

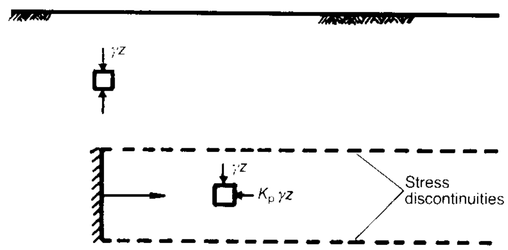

Fig. 8. Lower bound solution for a strip anchor pulled horizontally

where W_1 and W_2 denote the work done against the soil's weight in the two regions of the failure mechanism.

It is possible to envisage alternative more complex mechanisms but the two examined are considered to yield results close to the minimum upper bound. When evaluating upper bound solutions it is necessary to calculate the minimum solution for a given mechanism to determine as close a bound as possible to the theoretical failure loading. The minimization routine adopted in the computer program to analyse the upper bound solutions was the iterative method of Nelder & Mead (1965).

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Equation (2) is considered a poor lower bound solution for a strip anchor and, as shown in Fig. 11d, it does not compare favourably with the test results. However, as H/B tends to 1 the upper and lower bound solutions tend to converge, and in this condition equation (2) is an exact solution for the retaining wall problem. However, there are difficulties in altering the stress field of Fig. 8 to determine a more acceptable lower bound solution for an inclined strip anchor.

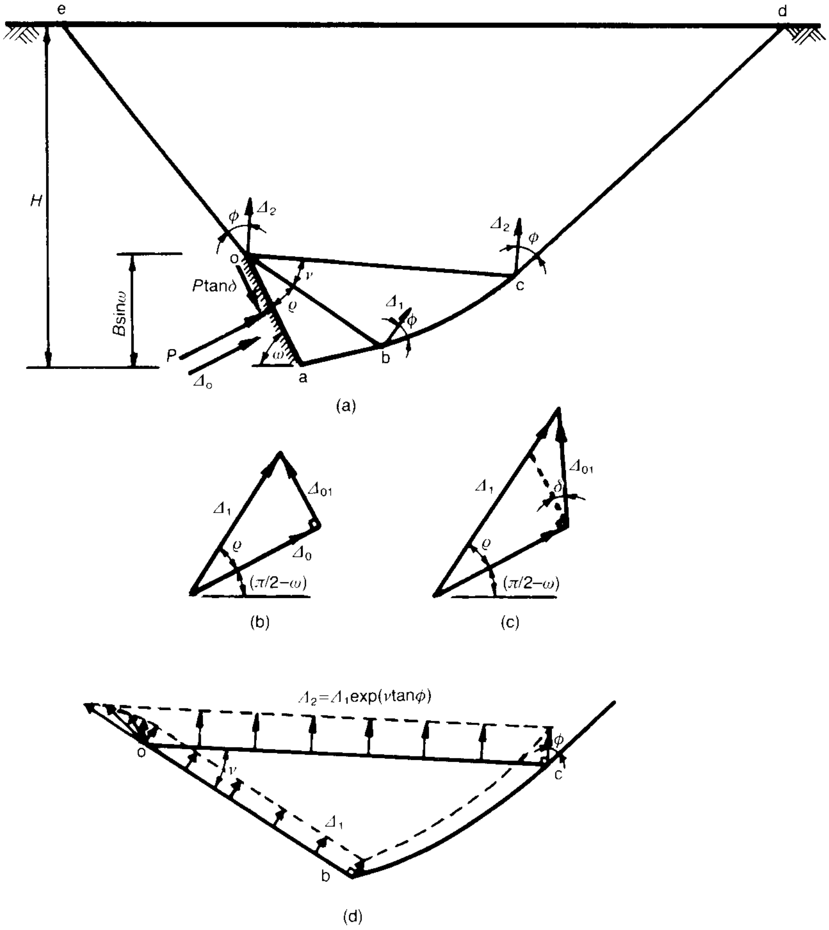

Fig. 9. Upper bound mechanism 1: (a) overall scheme; (b) vector displacement diagram for δ = 0; (c) vector displacement diagram for δ > 0; (d) log spiral zone obc

Theoretical upper bound solutions of mechanisms 1 and 2, as shown in Figs 9 and 10 respectively, have been prepared for $\delta = 0°$ and $\delta = 10{\cdot}6°$, and these are compared with the experimental results for plates of $L/B = 5$ and 10 at different angles of loading in Fig. 11a–d. The value of $\delta = 10{\cdot}6°$ is the peak friction angle between a polished steel plate and sand as measured in the direct shear box.

The results show that the upper bound solutions, particularly those involving an interface friction angle δ similar to that measured, give predictions of failure load reasonably close to the experimental values for H/B values of about 4 or less. For higher values the results diverge and in general the upper bound solutions tend to underestimate the experimental values. However, overall the upper bound solutions are considered to yield reasonable results for a true strip anchor. This is surprising because an upper bound solution for a material deforming under an associated flow rule is also an upper bound for a material deforming under non-associated flow conditions. A real soil such as that used in the experiments is probably more accurately described by a non-associated flow rule where $\psi < \phi$. This is discussed further when the applicability of the limit analysis solutions is assessed.

A comparison of the results for the two mechanisms indicates that as ω increases, mechanism 2 yields slightly smaller values of $P/\gamma AH$ than the more complex mechanism 1. Mechanism 1 does not reduce to the simpler mechanism 2 because of the restrictions on the displacement direction associated with the log-spiral failure zone. However, the results for the two mechanisms tend to converge at smaller values of ω and the failure configurations tend towards straight line solutions. Although it is obviously possible to imagine other forms of failure configuration, it is

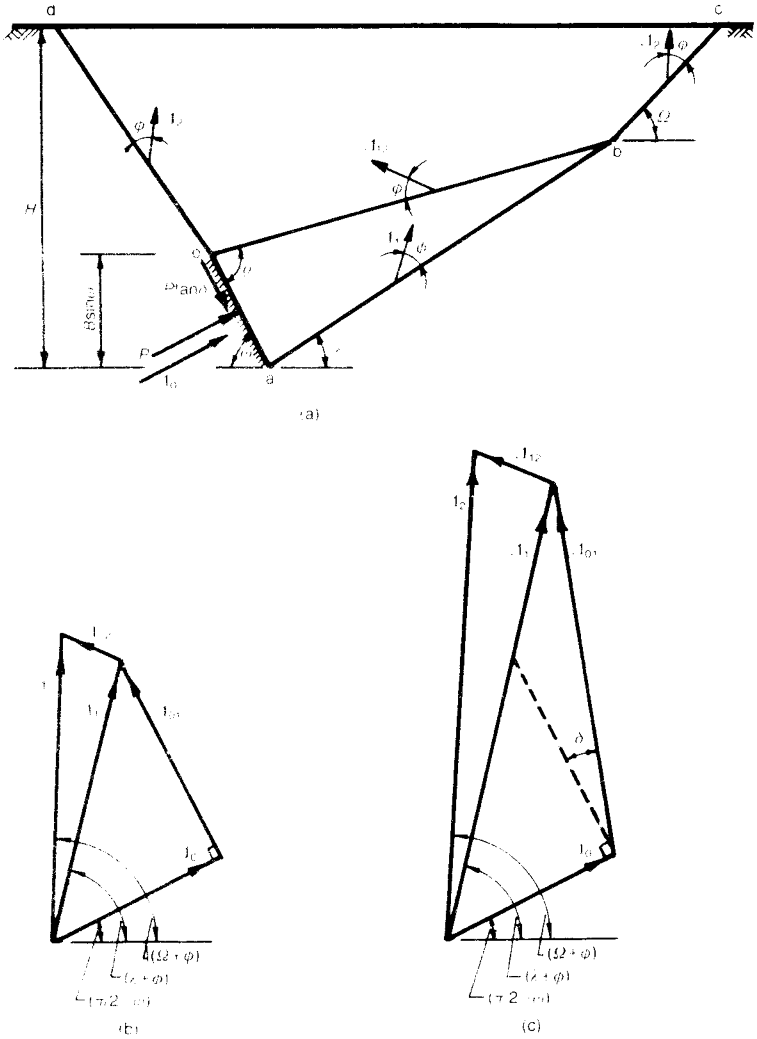

Fig. 10. Upper bound mechanism 2: (a) overall scheme; (b) vector displacement diagram for $\delta = 0$; (c) vector displacement diagram for $\delta > 0$

suggested that at angles of ω from 0° to 45° a mechanism that comprises straight lines which emanate from the plate edges provides a reasonable lower limit for an upper bound solution. A feature of the results for $\delta = 0$ is that the absolute minimum solution does not occur at $\omega = 0$ but takes place between $\omega = 0$ and 45°. This does not occur when δ is increased and in this case the minimum solution is for the vertical uplift of the anchor.

It is worth comparing the upper bound approach for the horizontal translation of strip anchors with the experimental results and the theoretical solutions of Neely *et al.* (1973) as shown in Fig. 12. In their analyses the stress characteristic configuration is determined from stress boundary conditions, assuming in one case a surcharge loading and in the other case an equivalent free surface loading. The latter form of loading can be considered to possess a varying degree of shear mobilization ($1 \geq m \geq 0$ where m is given by $s_0 = p_0 m \tan \phi$ and s_0 and p_0 are the shear and normal stresses on the equivalent free surface), thus producing a range of failure loads. Neely *et al.*'s results are for anchor plates embedded in a medium dense sand and Fig. 12 shows that, provided the interface friction angle is taken into account, there is good agreement between the upper bound solutions, the experimental results, and the stress characteristic solutions based on the equivalent free surface loading. Such correlation is perhaps particularly surprising in

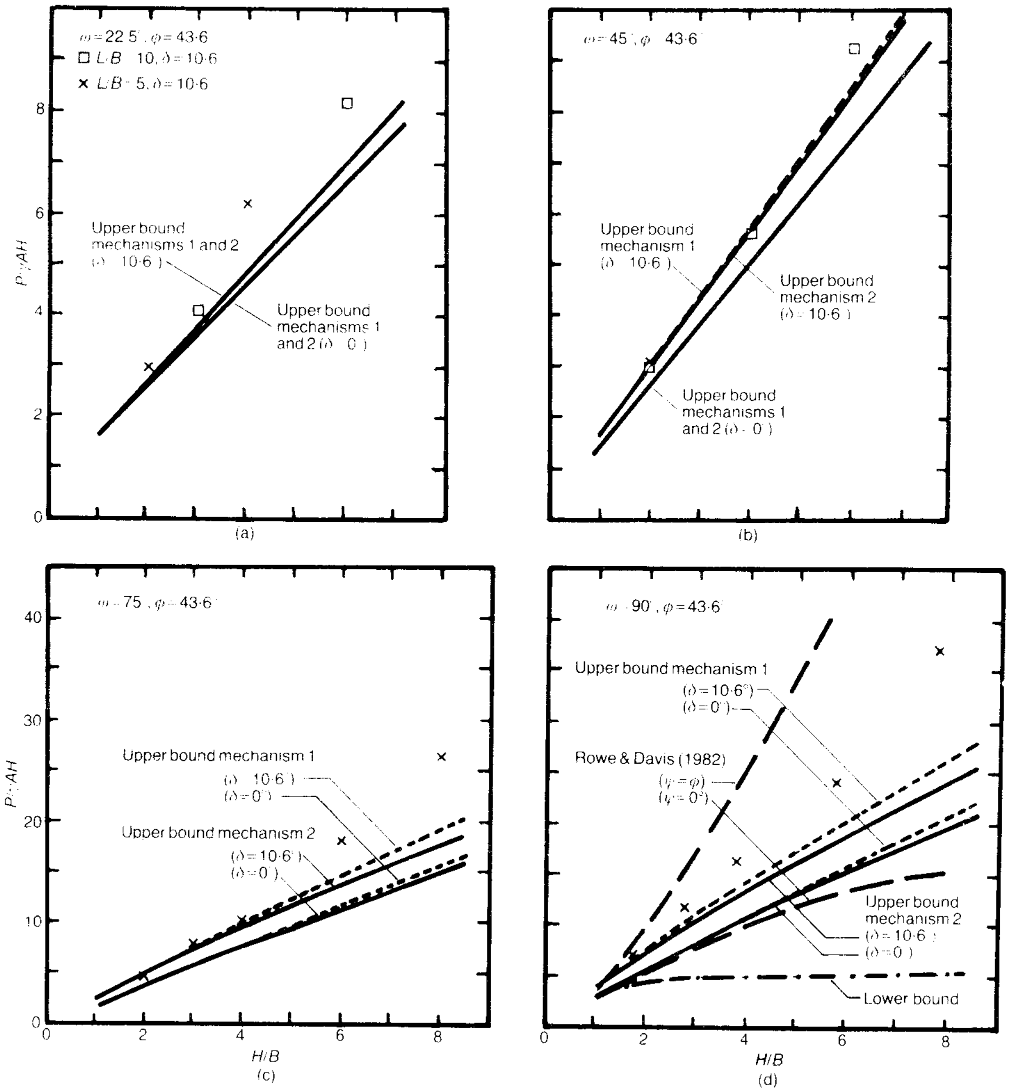

Fig. 11. Comparison of experimental and theoretical results: (a) for ω = 22·5°, φ = 43·6°; (b) for ω = 45°, φ = 43·6°; (c) for ω = 75°, φ = 43·6°; (d) for ω = 90°, φ = 43·6°

medium dense sand because it will exhibit less dilatancy than dense sand and thus there is a greater difference in behaviour between the actual soil and the idealized soil assumed in the limit analysis solutions.

Rowe & Davis (1982) have adopted a finite element approach to determine the load–displacement response of strip anchors pulled horizontally through an elastic plastic frictional soil. They have examined the effects of soil dilatancy ψ and initial stress state K_0 as well as the difference in behaviour between perfectly rough and perfectly smooth anchor plates. Careful selection of values for the parameters allows a close correlation with the experimental results of this Paper, and good agreement is given with $\phi = 43{\cdot}6°$ and $\psi = 18°$, the effect of interface friction and initial stress level being relatively small over most of the depth range. However, two cases of particular interest have been examined in Fig. 11d, where predictions are presented of $P/\gamma AH$ values against H/B for a soil obeying an associated flow rule ($\psi = \phi$), as is assumed in the limit analysis solutions, and for a non-dilatant soil ($\psi = 0°$). A smooth anchor plate has been assumed. It may be noted that the upper bound approach yields results which are lower than those obtained by Rowe & Davis's analysis with

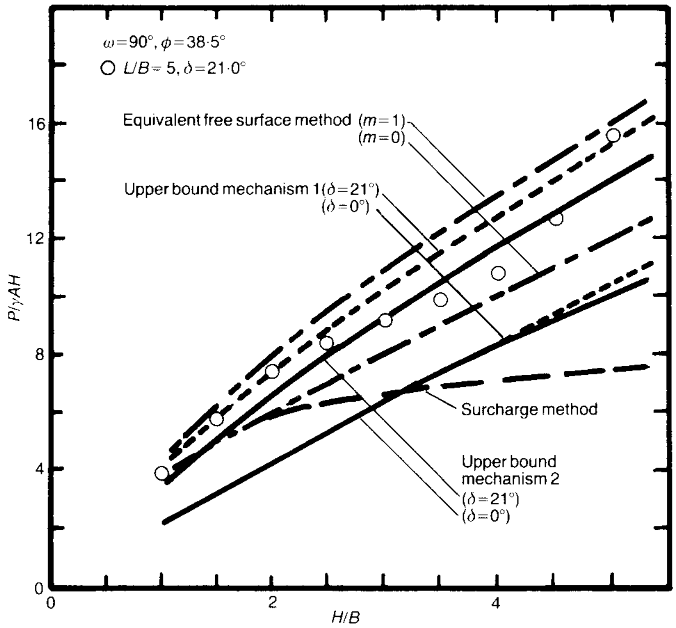

Fig. 12. Comparison of experimental and theoretical results of Neely *et al.* (1973) with the upper bound limit analysis solutions

$\psi = \phi$, but which exceed the results for the case $\psi = 0°$. The plots indicate the sensitivity of their analysis to the assumed angle of dilation. Values of $\psi = 5°–12°$, which would be likely to encompass the average values activated in practice, give $P/\gamma AH$ results comparable to those of the upper bound solutions.

APPLICABILITY OF LIMIT ANALYSIS SOLUTIONS

The concept of the limit theorems embodies the fact that no energy is dissipated by friction when $\psi = \phi$, as the direction of dilation is perpendicular to the frictional force. This is not the case observed in real soils where ψ is less than ϕ. Soils tend to fail in a progressive manner with the mobilized strength and dilation of the soil varying throughout, with some regions having already experienced their peak condition while other regions have not yet attained it. The effect of this on anchors pulled horizontally has been discussed by Dickin & Leung (1985). It is assumed in the limit analyses that the peak angle of friction as determined in a direct shear box is applicable and that the angles of friction and dilation are constant on all failure surfaces and in all failure regions.

The proof of the limit theorems is based on the principle of virtual work which requires that changes in the geometry of the problem up to the limit load are insignificant. The limit theorems are applicable as long as it reasonable to use original undeformed dimensions in the equilibrium equations. Although displacements at failure load are small, for anchors inclined to the vertical, the non-uniform distribution of stress on the anchor face results in a deviation from the assumption of pure translation, which is considered to account to a great degree for the upper bound limit analysis solutions' generally underpredicting the experimental data. Shaft friction on the tie rod, and its rigidity and connection with the anchor plate, are also likely to play a part although to what extent is uncertain.

Figure 13a–d compares the predicted volumes of sand involved in resisting anchor movement, based on the upper bound mechanisms 1 and 2, with the observed extent of surface displacements and the positions where rupture planes exited at the surface, for the plates of $L/B = 5$ and 10. It is obvious that the theoretical mechanisms, although kinematically acceptable for the idealized material obeying an associated flow rule, do not provide realistic failure patterns in the real sand. The extent of the predicted surface movement is greater than the actual movement. As indicated by the downward arrows, slippage occurred behind the anchor plates at angles of loading from $\omega = 45°$ to $90°$, although in general only in the case of $\omega = 90°$ was it measurable at the sand surface before peak loading. It is instructive to compare the measured surface movements and predicted failure patterns given by the upper

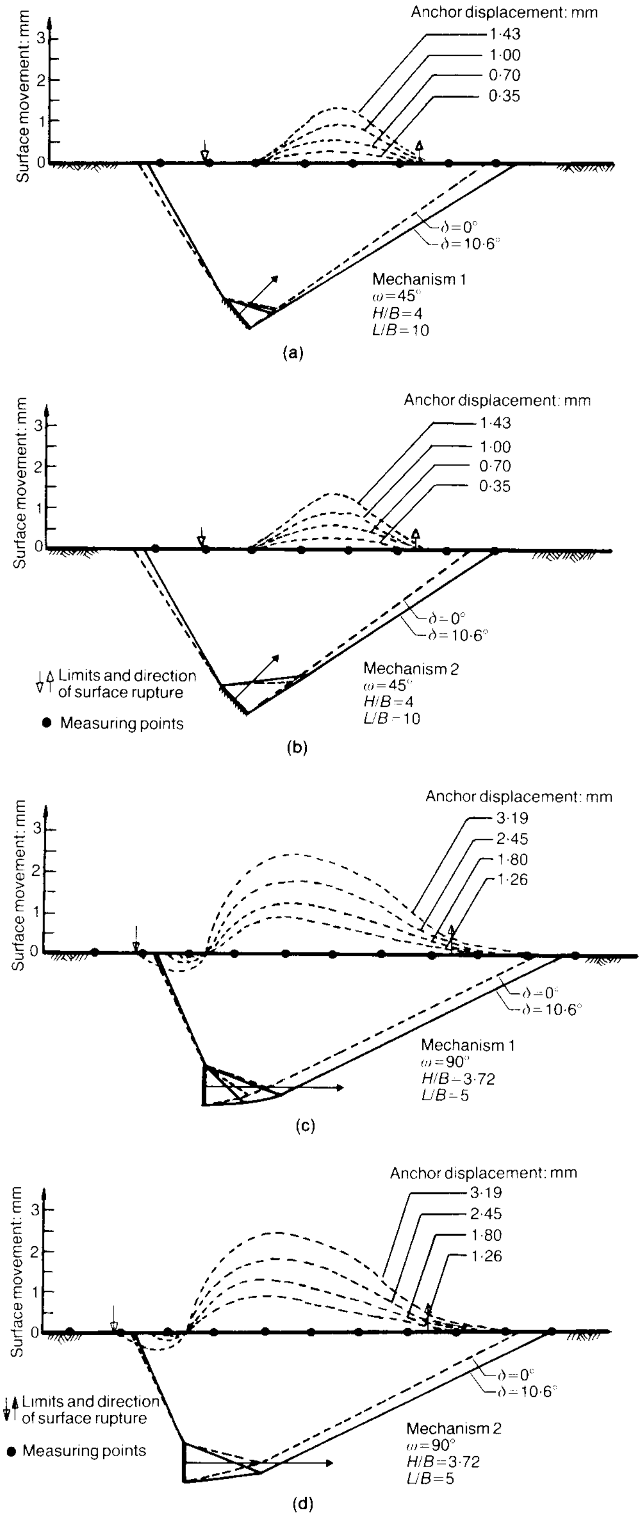

Fig. 13. Predicted and experimental sand movement: (a) mechanism 1, ω = 45°; (b) mechanism 2, ω = 45°; (c) mechanism 1, ω = 90°; (d) mechanism 2, ω = 90°

bound mechanisms with the predictions of the finite element method of Rowe & Davis (1982) which show the increased extent of the sand involved in shearing with increased angle of dilation. The experimental results suggest that the mass of sand involved is not as great as predictions would indicate.

SUMMARY AND CONCLUSIONS

The experimental results presented for the ultimate passive resistance of anchor plates in very dense sand have been plotted in terms of the dimensionless parameters $P/\gamma AH$ and H/B. It is shown that to a reasonable approximation the shape factor defined by equation (1) is constant for all angles of loading; at least for H/B values in excess of about 2. The ultimate passive resistance and corresponding displacements at failure for a given plate are shown to increase with depth and angle of loading, the greatest changes occurring within the interval $\omega = 45°$ to $\omega = 90°$.

Upper and lower bound limit analysis solutions, assuming a soil obeying an associated flow rule ($\psi = \phi$), have been developed for the case of a strip anchor ($L/B = \infty$). The theoretical solutions are compared with the experimental results for plates of $L/B = 5$ and 10, which are considered to give a reasonable guide to the behaviour of a true strip anchor. A poor lower bound solution is formulated. Upper bound solutions are shown to yield results for ultimate passive resistance which compare favourably with the experimental evidence, when the interface friction between sand and the anchor plates is taken into account. However, the failure configurations cannot be taken to represent the actual sand movements.

Comparison is also made between the upper bound solutions and the experimental results and equivalent free surface stress characteristic solution of Neely *et al.* (1973), for horizontal translation of anchors in medium dense sand. Such a soil is likely to exhibit less dilation than a dense sand and thus compare less favourably with the theoretical assumption $\psi = \phi$. However, there is again good agreement if interface friction is taken into account in the upper bound solutions.

The finite element approach of Rowe & Davis (1982) for strip anchors pulled horizontally through an elastic-plastic soil has also been examined. For a soil that obeys an associated flow rule, $\psi = \phi$, the results of Rowe & Davis's theoretical solution greatly exceed those given by the upper bound limit analysis approach.

Based on the evidence, the upper bound solutions may be used to assess the ultimate passive resistance of inclined anchors in dry sand. It would be a relatively easy matter to extend the upper bound solutions to cover the cases of anchor plates of finite length and to soils that possess internal friction and cohesion. The results do not cater for the possible effects of repetitional, cyclical or dynamic loading.

APPENDIX 1. METHOD FOR DETERMINATION OF UPPER BOUND SOLUTIONS

Consider mechanism 1 with $\delta > 0$ (Fig. 9). The directions of Δ_1 and Δ_2 are controlled by the angles ρ and v respectively and the directions of displacements are perpendicular to the lines ob and oc. As the medium under consideration is cohesionless, the external work done in any region is the self-weight multiplied by the vertical component of displacement.

Triangular zone oab

$$\frac{W_1}{\gamma BH} = -\frac{1}{2}\frac{B}{H}\Delta_1 \cos(\omega - \rho) \times \sin\rho \cos\rho\,(1 + \tan\rho \tan\phi) \qquad (5)$$

Log-spiral zone obc

The zone obc consists of failure lines made up of concurrent straight lines and logarithmic spirals. Chen (1975) has shown that such a zone forms an acceptable mechanism, all displacements being uniform and perpendicular to the radial lines. The external work done by the soil weight in this zone can be computed by summing over the region v the products of the weight of all smaller triangles and the triangles' vertical component of displacement. This leads to

$$\frac{W_2}{\gamma BH} = -\frac{1}{2}\frac{B}{H}\Delta_1 \cos^2\rho\,\frac{(1 + \tan\rho\tan\phi)^2}{(1 + 9\tan^2\rho)} \times [3\tan\phi\exp(3v\tan\phi)\cos\alpha - \exp(3v\tan\phi)\sin\alpha - 3\tan\phi \times \cos(\omega - \rho) + \sin(\omega - \rho)] \qquad (6)$$

Zone ocde

$$\frac{W_3}{\gamma BH} = -\Delta_1\exp(v\tan\phi) \times \left\{D\left[1 - \frac{B}{H}\sin\omega + \frac{1}{2}\frac{B}{H}D\sin\alpha\right] + \frac{1}{2}\frac{H}{B}\tan\phi\cos\alpha\left[\left(1 - \frac{B}{H}\sin\omega\right)^2 \times (1 - \tan(\phi - \alpha)\tan\alpha) + \left(1 - \frac{B}{H}\sin\omega + \frac{B}{H}D\sin\alpha\right)^2 \times (1 + \tan(\phi + \alpha)\tan\alpha)\right]\right\} \qquad (7)$$

in which $D = \cos\rho\,(1 + \tan\rho\tan\phi)\exp(v\tan\phi)$ and $\alpha = (\omega - \rho - v)$.

The limit theorems are not strictly applicable for a plate with interface friction. However, Drucker (1954) states that 'any set of loads which will not cause collapse of an assemblage of elastic–plastic bodies with frictional interfaces will not produce collapse when the interfaces are "cemented" together with a cohesionless soil of friction angle $\phi = \delta$'. Thus by allowing dilation on the interface oa such that the displacement is inclined at an angle δ to this surface, the calculated collapse load will be an upper bound for a material with a frictional interface.

From Fig. 9b

$$\Delta_0 = \Delta_1 (\cos \rho - \tan \delta \sin \rho) \tag{8}$$

Thus equating the work done by external forces, (i.e. by P and the soil's weight) to the dissipation of energy (in this case zero), gives

$$P\Delta_0 = -W_1 - W_2 - W_3$$

$$\frac{P}{\gamma BH} = -\frac{(W_1 + W_2 + W_3)}{\gamma BH\Delta_1} \cdot \frac{\cos \delta}{\cos (\rho + \delta)} \tag{9}$$

The expressions for W_1, W_2 and W_3 are substituted into equation (9) to obtain the upper bound solution. It is then necessary to evaluate the minimum $P/\gamma BH$ for given conditions. This is best achieved by selecting a suitable computer minimization routine. Similar procedures may be used for other postulated geometries.

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