Toll, D.G. and Ong, B.H. (2003). Critical state parameters for an unsaturated residual sandy soil. Geotechnique 53 (1), 93-103

Discussion by:
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The paper deals with the critical state of a residual soil, Jurong soil, but also includes previously published data for another residual soil, Kiunyu gravel (Toll, 1990). The interpretation of the data is based on the premise that there is a smooth transition between unsaturated and saturated behaviour. This may be true and seems a reasonable hypothesis at first consideration but relies on the progressive breakdown in the aggregated structure associated with unsaturated conditions to the more dispersed condition associated with saturated soils. However, an alternative philosophy might argue that if the suction in an unsaturated soil is sufficient to maintain the structure at critical state it is likely that it will hold everywhere. Only at relatively low suctions when the structure can no longer be maintained under shearing will it break down and at the critical state is likely to break down everywhere. A smooth transition may thus not occur. This is important in that as rightly identified by the authors and Toll (1990), the behaviour under a suction-controlled aggregate structure can be expected to differ from that of a dispersed structure.

Murray (2002) and Murray et al (2002) describe the derivation and use of the average volumetric 'coupling' stress' p'_c for unsaturated soils as given below:

$$p'_c = (p-u_a) + s\, v_w/v \qquad (1)$$

Where, p is the mean total stress
- $s = (u_a-u_w)$
- v_w is specific water volume
- v is specific volume
- u_a is pore water pressure
- u_w is pore water pressure

Normalised plots of q/s against p'_c/s for the Jurong soil and Kiunyu gravel are presented in Figs 1 and 3. The data fall on relatively straight lines with an intercept on the q_c/s axis at $p'_c/s = 1$ in both cases close to 0.6. As shown by Murray (2002), if the critical state data of Wheeler and Sivakumar (1995) for kaolin is plotted in a similar manner, this also results in a straight-line relationship with a similar intercept close to 0.60 on the q/s axis.

Adopting the nomenclature of Toll and Ong, the equation of the straight line is given by,

$$\frac{q}{s} = M_a \left[\frac{p'_c}{s} - 1\right] + \Lambda \qquad (2)$$

where, M_a is the total stress ratio

Λ is the intercept on the q/s at $p'_c/s = 1$

Using equation (1), this may be written in a similar form to the deviator stress equation (4) in the paper of Toll and Ong as,

$$q = M_a (p-u_a) + M_b s \quad (3)$$

where, M_b is the suction stress ratio and is given by,

$$M_b = M_a \left[\frac{v_w}{v} - 1 \right] + \Lambda \quad (4)$$

Figures 2 and 4 show plots of M_b against v_w/v for the Jurong soil and Kiunyu gravel using equations (3) and (4) and taking $\Lambda = 0.6$. The plots again suggest a linear relationship as has been found for kaolin (Murray, 2002).

Figures 1 to 4 indicate a consistent interpretation of the data with M_a constant and M_b decreasing with decreasing v_w/v (increasing suction).

For a saturate soil at critical state $q = M p'$ (where M is the critical state stress ratio and p' is the effective stress). There is no obvious indication of a smooth transition between unsaturated and saturated conditions. For the Jurong soil M =1.23 and M_a is interpreted as approximately 1.27. For the Kiunyu gravel M = 1.62 and M_a is 1.76. From the results of Wheeler and Sivakumar (1995) on kaolin M = 0.82 and $M_a = 0.86$. $M_a > M$ is interpreted as implying the creation of aggregated packets of particles that act as 'large particles' (Toll, 1990). Once created, however, the experimental evidence suggests that the modified soil structure results in a relatively constant M_a unaffected by changes in suction. The change in suction influences M_b. The formulation thus separates the influence of suction from the influence of soil structure. M_b decreases with decreasing v_w/v consistent with equation (4) and the water phase being drawn back into the finer pores within the soil packets and having a reducing influence on the interpacket contacts where shearing is taking place (Toll, 1990).

Murray (2002) also shows that the normal consolidation lines for kaolin (Wheeler and Sivakumar, 1995) may be represented by the following equation:

$$v = N_t - \lambda_t \ln [(p-u_a) + s\, v_w/v] \quad (5)$$

where, N_t is the extrapolated value of p'_c at 1.0kPa

λ_t is the slope of the straight-line portion of the consolidation plots

The equivalent critical state lines would be given by,

$$v = \Gamma_{ab} - \lambda_a \ln [(p-u_a) + s\, v_w/v] \quad (6)$$

where, Γ_{ab} is the extrapolated value of p'_c at 1.0kPa
λ_a is the slope of the critical state line

Toll and Ong indicate the following 5 variables as necessary to describe the critical state for unsaturated soils:

q, $p-u_a$, s, v and S_r

where S_r is the degree of saturation

They also indicate that five parameters are required,

M_a, M_b, Γ_{ab}, λ_a and λ_b

However, the forgoing formulation suggests that v_w or v_w/v is probably a more appropriate volumetric variable than S_r and it may be possible to reduce the number of parameters from 5 to 4. The suggested changes arise from equation (4), which indicates $M_b = f(M_a, v_w/v, \Lambda)$ and equation (6), which suggests that λ_b is unnecessary. The significance of v_w/v is that it represents the volume of the saturated 'packets' per unit volume of soil.

That is, the following 5 variables may prove more fundamental in an understanding and interpretation of the critical state for unsaturated soils.

q, $p-u_a$, s, v and v_w (or v_w/v)

The following 4 parameters also emerge from the analysis,

M_a, Λ, Γ_{ab}, λ_a

It may be possible to further reduce the parameters to 3 if as for the Jurong soil, Kiunyu gravel and kaolin, Λ can be shown to be constant at 0.6 for other soils. In addition, importantly, M_a is shown to be relatively constant for a given soil. This means that in accordance with equations (3) and (4) if a relationship between s and v_w/v at critical state is established for a soil, only a limited number of strength determinations are necessary to determine M_a and Λ and facilitate strength assessments under other conditions. There is however, the possible discontinuity between the behaviour in the saturated and unsaturated states as a result of soil structural changes that needs further investigation.

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